Phases of a holographic QCD

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Based on

B. Gwak, MK. Kim, B. Lee, Y. Seo, SJS; arXiv:1105.2872
Plan

• 0. QCD and String
• I. gauge/gravity duality
• II. confinement in hQCD

• III. Phases of a hQCD
0. QCD and string

1. Low energy spectrum: Regge trajectory $\rightarrow$ string spec.
2. Confinement $\rightarrow$ Flux string

- Motive of string theory 40 years ago.
- Gone with the asymptotic freedom.
- String scale is identified with gravity scale rather than QCD scale.
Why string theory for qcd Now?

• Shear viscosity /entropy density in RHIC (Son et.al)

• experiment:  $< 0.1$

\[ \eta/s = \frac{1}{g^4 \ln g} \gg 1 \quad ; \quad \text{ads/cft: } \sim \frac{1}{4\pi} = 0.09 \]

• How: scaling out string scale.

• Fact: the result is for deconfined phase

• Q: Confinement and Hadron physics?
1. gauge/gravity duality

- 1. Holographic: 5d gravity theory for 4d QCD

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**Holographic principle**

Many-body system without gravity in $d$ spacetime dimension

= quantum gravity in $d+1$ dimensions

$z = 0$

UV

RG scale

IR

Organizing principle: UV/IR connection

Fig. from Hong Liu
Validity of gravity regime

• gravity limit is valid if string looks pointlike.
  \[ \lambda = g^2 N_c >> 1 \]

No loop necessary.

1. Ls<<Rads
2. gs<<1

Large N theory. Good and Bad
Relevant to qcd?

• Super-symmetric version of qcd
  Original version is N=4 SUSY. Without quarks SUSY can be broken by BC. T. d. etc

• Just a model or effective theory to be compared with experiment.
  Some results are too good to be irrelevant ...... [eta/s, gluon mass, ads/qcd, SS]
Dictionary

- Let $O(x)$ is an color singlet operator with dimension $\Delta$ and spin $p$
  $A(x)$ is an source of it.

- Then AdS/CFT says:
  Extend it to $d+1(=5)$ dim by

$$A(x, z) = A(x) z^{d-p-\Delta} - \langle O \rangle z^{\Delta-p} + \ldots.$$ 

- If we know action and BC, exact 2pt function can be calculated \text{CLASSICALLY.}
Ex: Density and chem. Pot.

• Holographic Image of baryon number is a LOCAL U(1) charge $\rightarrow$ ‘electric’ potential $A_0$.

• In 5d,

$$A_0 = \mu z^{3-\Delta} - Q z^{\Delta-1}$$

$\Delta = \text{dimesion of Operator } = 3$

$Q = \langle \bar{\psi} \gamma_0 \psi \rangle$

$\mu = \text{chemical potential}$
Gluon condensation and dilaton

- \( \text{Tr}(FF) \) is dual to scalar \( \phi \)
- Extend it to 5-dim by
  \[
  \phi(x, z) = \phi_0 z^{4-\Delta} - cz^\Delta
  \]
  \[
  \phi_0 = 0 \quad c = \langle Tr F^2 \rangle
  \]
Chiral condensation and mass

- Mass op. $\bar{\psi}\psi$ is dual to a scalar $\sigma$
- Extend $\sigma$ to 5 dim by

$$\sigma(x, z) = m_q z - cz^3$$

$$c = \langle \bar{\psi}\psi \rangle$$
Wilson loop

\[ W(\mathcal{C}) = \frac{1}{N} Tr P e^{i \oint_\mathcal{C} A} \]

\[ \langle W(\mathcal{C}) \rangle = A(L) e^{-T E(L)} \]

\( E(L) \) is the energy of the quark-antiquark pair.

Fig from Malacena: 9803002
hQCD = deformation of N=4

Qm Gluon dynamics $\rightarrow$ classical Geometry.

Confinement by geometry.

Mass gap by integration of motion

Flavor dynamics by classical fields in warped geometry.
State(bd)/Geometry(Bk) correspondence

• Ground state of N=4YM / AdS geometry
• Excitation of YM / Deformation of AdS

Ex: temperature / BH
  Confinement / IR cutoff by hand
  or by repulsive gravity
  Ex: double wick rot. of BH

By UV/IR, cut-offed ads is dual to Bag.
YM with Temperature $\leftrightarrow$ BH

Finite temperature

Boundary

Event horizon

$z = 0$

boundary system at a finite temperature

$z = z_0 \sim T$

Black hole

Entropy, energy, Euclidean action of BH

Entropy, energy, free energy of YM theory
BH is the Geometry with

Screening of quarks in a QGP

(3+1)-dim world at temperature $T$,

Quarks are screened

$N=4: \quad L_s = 0.277/T$, \quad QCD (2 flavor): \quad L_s \sim 0.5/T$

(lattice)
Geometry with repulsive net force $\rightarrow$

Confinement

(3+1)-dim world

Spacetime terminates here smoothly

Zm
Confinement [Gluon v.s Quark]

• For gluon: mass gap with discrete glue-ball. only IR cut off is important.
• For quark confinement: \( <W> \sim \text{Area Law} \)
  Hard wall, repulsive core:
  In most cases, two agrees but there are exceptions (later)
• BH \( \rightarrow \) Deconfined phase
• Meson: (Light) D-brane vibration
  (Heavy) Long string.
Basic picture for players

- **Baryon**: compact D5+Nc strings Dynamics:
- **Multi-baryon**: Homogenous smeared-out BV along D4 direction
Problem: Phase diagram

Confinement/deconfinement
Chiral tansition
Conf/deConf Phase transition

- Hawking Page Transition, i.e., If there is a scale at T=0, e.g. KK, Two geometries can compete.
- $F[G_1] < F[G_2] \Rightarrow G_1$ is dominating the path integral.
- First order phase tr. Not good for QCD.
- All physical quantity is discontinuous.
Two puzzles in hQCD

• Temperature dependence of physical quantity in Confined phase.

• Chiral condensation increases with density, which is opposite to the field theory expectation.
The origin of puzzle

• Hawking Page transition: When there is a scale other than the temperature, there are competing geometries. (Ex: ads BH v.s thermal ads)

• The geometry for the Low temperature phase does not have any temperature dependence. Flat direction is c’pt.
To overcome this difficulty.

• Need geometry that does not allow any HP transition
  Geometry w/o KK, w/o hardwall.

• Question is how to obtain confinement, which need a scale!
  And a scale means HP!

• confining BH?
Hadrons without confined gluons (glueball)

- possibility that there is no gluon confinement nevertheless hadrons are allowed.

Not surprising to have Mesons in BH by flavor branes (See figure)

- Baryons as well as Mesons!
Mesons and baryons in the black hole phase.

• For the meson, it is known to exist in the black hole phase unless there are free quarks. This is due to the presence of the black hole embeddings.

• However, usually the baryons are allowed simply because compact D5 (Witten baryon vertex) can not be sustained in the sky of the black hole.
The idea: gluon condensation

• It break the scale symmetry therefore introduces a scale.

\[ \partial_\mu T^{\mu\nu} \sim \langle \bar{\psi}\psi \rangle + \langle Tr F^2 \rangle \]

• Dual of gluon Op = dilaton, a scalar.
• But if dilaton has vev, usually 5d vac is singular with repulsive core.
• Can one find a regular geometry with vev of scalar?
\[ ds_{10}^2 = e^{\Phi/2} \left[ \frac{r^2}{R^2} (f(r)^2 dt^2 + dx^2) + \frac{1}{f(r)^2} \frac{R^2}{r^2} dr^2 + R^3 d\Omega_5^2 \right], \]

\[ e^\Phi = 1 + \frac{q}{r_T^4} \log \frac{1}{f(r)^2}, \quad \chi = -e^{-\Phi} + \chi_0, \]

\[ f(r) = \sqrt{1 - \left(\frac{r_T}{r}\right)^4}, \]

where \( R^4 = 4\pi g_s N_c \alpha'^2 \) and \( q \) is a (vev) of gauge fields condensation.

number of D-instanton.
Character of geometry

- No KK, \( \rightarrow \) NO Hawking-Page PT
- Conformally Black D3.
- Dilaton is Singular at horizon.
- Finiteness of bulk action: cancellation between dilaton and axion.
- Finiteness of DBI action: detailed str.
Effect of $q$:

**Pseudo confinement**

Effect is there since string action in string frame.

i) confinement at 0 temperature.
   at large enough separation,
   ii) linear potential before critical separation where it is screened.
baryons exist why? repulsive nature of q

- D-brane dynamics is given by DBI action which has dilaton factor. Also it is calculated in the string metric. So it is affected by the presence of q.
- BV allowed due to the repulsion between D-1 /D5

\[
S_{D5} = -\mu_5 \int e^{-\Phi} \sqrt{-\text{det}(g + 2\pi\alpha' F')} + \mu_5 \int A_{(1)} \wedge G_{(5)}
\]

\[
= \tau_5 \int dt d\theta \sin^4 \theta e^\Phi \left[ -\sqrt{\frac{e^{\Phi} \omega^2}{\omega_+} (\xi^2 + \xi'^2) - \tilde{F}^2 + 4\tilde{A}_t} \right]
\]
q allows baryon!

**Figure 12:** (a) $\xi_0$ dependence of D5 brane embeddings with $q = 10$, $\xi_T = 0.5$ (b) $q$ dependence of tip of D5 brane($\xi_c$).
Chiral dynamics

$q$ dependence of chiral condensation is consistent with field theory result.

\[
\langle \bar{\psi} \psi \rangle = \frac{\alpha_s N_f}{12\pi m_q} \langle \text{Tr} F^2 \rangle
\]
Geometric understanding of chiral dynamics

Figure 1: (a) D7 brane embeddings for $m_q = 0$ with $q = 0, 0.1, 0.3, 0.5, 1, 3, 5, 10$ from bottom to top. (b) D7 brane embeddings for $m_q = 5$ with $q = 0, 0.1, 0.3, 0.5, 1, 3, 5, 10$ from bottom to top.
Chiral symmetry breaking in quark phase

$\xi_T=1.0724$, $q=20$, $Q=1$
Understanding the chiral condensation plot

\[ \xi_T = 1, \hat{Q} = 0.1 \]

\[ \xi_T = 1, \hat{Q} = 10 \]
Baryon Phase: chiral symmetry is always broken.
Density plot of chiral condensation
Phase diagram

$q = 50$

Quark Phase ($\chi_S$)

Quark Phase ($\chi_{SB}$)

Baryon Phase ($\chi_{SB}$)
Conclusion

• Confinement and chiral transition separate.
• We can have temperature dependence in hadron phase.
• Decreasing chiral condensation in deconfining phase as function of density. All the bottom-up model so far shows the opposite behavior.