A gravity dual of Hadronic dense matter

Sang-Jin Sin (Hanyang) @SKKU 2009/06/18

Based on
SJS , 0707.2719
CY.Park, BH.Lee, SJS 0905.2800
X.Ge, Y. Matsuo, Shu, S.Takeuchi, T. Tsukioka, C.Yoo , archiv:0901.0610,
0806.4460
Nuclear Physics

has two well known difficulties:

* coupling is large

* many particle system
Fundamental theory is known to be QCD

Asymptotic freedom $\rightarrow$ solvable in high $E$.

However, in low Energy regime
Now $\sim$ 40 years ago.
What can be done?

- Introduce new degree of freedom whose interactions are weak.

Duality
• Open st. bubble /Closed st. propagator

→ quantum gauge / classical gravity

→ AdS/CFT correspondence.
Consequences of duality

- Gluon dynamics is replaced by ads gravity.
- Correlation function in 4d can be calculated by the classical dynamics at the ads bulk.
Phases of QCD
AdS/CFT at finite $T$
AdS/CFT for Hadron
II. Hadron

1. Confinement mechanism
2. Meson and Baryon
3. Dense matter
4. Thermodynamics
Confinement

• In QCD, $\Lambda_{\text{QCD}}$ is introduced by dynamics.

• In hQCD it should be encoded by BC.

→Hard Wall, Bubble, Mkk
Meson and Baryon

• Meson is vibration of D-brane → Probe brane

• Baryon is a combination of Baryon vertex + Nc strings
Dense matter

• Quark is the end point of strings ending on the probe brane.

• Chemical potential is the work to introduce a particle against 5d electric fields created the charge.

\[ \mu = \int F \_0 r \, dr \]
Geometry for the Dense matter?

- Bulk Filling brane
- \( U(1) \) for the Brane charge
  \( U(1) \) for Bulk R charge
  \( \rightarrow \) They have the same coupling to gravity.
- Charged black hole can be used to describe the QGP.
QGP: Charged BH

\[ S_M = \int d^5 x \sqrt{-G} \left[ \frac{1}{2\kappa^2} (R - 2\Lambda) - \frac{1}{4g^2} F_{MN}F^{MN} \right] \]

\[ A_0 = A_0(z), \]
\[ A_i = A_i = 0 \quad (i = 1, \cdots, 3), \]
\[ ds^2 = \frac{R^2}{z^2} \left( -f(z)dt^2 + dx_i^2 + \frac{1}{f(z)}dz^2 \right) \]
\[ f(z) = 1 - mz^4 + q^2z^6, \]
\[ A_0 = \mu - Qz^2, \]
\[ \frac{1}{2\kappa^2} = \frac{N_c^2}{8\pi^2 R^3} \quad \text{and} \quad \frac{1}{g^2} = \frac{N_c N_f}{4\pi^2 R}, \quad Q = \sqrt{\frac{3}{2} \frac{N_c}{N_f}} q. \]
Phase Transition

• In conformal theory, No PT.
• Need a scale.
• Any confining theory has PT at Tc.
• Geometry should be paired for high and low T (g_confine, g_BH)
Hadron geometry?

\[ ds^2 = \frac{R^2}{z^2} \left( (1 + q^2 z^6) d\tau^2 + d\vec{x}^2 + \frac{1}{1 + q^2 z^6} dz^2 \right), \]

with IR cut-off \( z_{IR} \)

IT has singularity at \( z=0 \)
But that is in cutoffed regime.
What is the point?

- Metric depends on the density
  cf: Thermal ads
- We can study the density dependence of the Hadronic matters although temperature dependence is STILL Hard.
Boundary condition

Dirichlet boundary condition

\[ A(z_{IR}) = i\alpha \mu, \]

in the limit of \( \epsilon \to 0 \) the renormalized action of the tcAdS becomes

\[
\bar{S}_{tc}^D = -\frac{V_3 R^3}{\kappa^2} \frac{1}{T_{tc}} \left( \frac{1}{z_{IR}^4} + \frac{2\kappa^2}{3g^2 R^2} \frac{(1 - \alpha)^2 \mu^2}{z_{IR}^2} \right). 
\]

From this, the particle number \( N \) is given by

\[
N = \frac{2}{3} (1 - \alpha) \frac{2R}{g^2} Q V_3. 
\]
Relation of charge & chemical potential

$$\mu N = S_b T_{tc}$$

$$S_b = \frac{\mu}{T_{tc}} \frac{2R}{g^2} Q V_3,$$

$\Rightarrow \quad \alpha$ should become $-1/2$ for the consistency.

$$\mu = \frac{2}{3} Q z_{IR}^2.$$
Phase diagram

\[ \tilde{\mu}_c = \sqrt{\frac{3N_c}{N_f} \frac{1 - 2z_c^4}{z_c^2 (9z_c^2 - 2)}} \]

\[ \tilde{T}_c = \frac{1}{\pi z_c} \left( 1 - \frac{1 - 2z_c^4}{9z_c^2 - 2} \right) \]

\[ \bar{\mu}_B = 3\mu + m_B \]
Rho meson in dense medium

\[ \delta A_{\mu} = V_{\mu}(z, p)e^{ip \cdot x}. \]

\[ 0 = \partial_{z}^2 V_i - \frac{1}{z} \frac{(1 - 5q^2z^6)}{(1 + q^2z^6)} \partial_z V_i + m_m^2 V_i, \]
Exited mesons
III QGP phase

• Au-Au collision to make QGP at RHIC
  → quantum liquid
  → Hydrodynamics
Relativistic Heavy Ion Collider (Brookhaven N.L)

- Au-Au collision
- $E \approx 200$ GeV/nucleon
- Seek quark-gluon plasma (QGP)
Detectors

star

phenix
Liquidity due to strong interaction
puzzles

• elliptic flow (hep-th/0610113).
• Perfect fluid: (0806.4460, 0901.0610)
• Jet quenching (hep-th/0607123)
• Early thermalization (hep-th/0511199)
• These are puzzles from the aspect of AF but clues to use AdS/CFT.
Perfect fluid

Exp: \( \frac{\eta}{s} < 0.1 \times \frac{\hbar}{k_B} \)

While perturbative evaluation \( \sim 1/g^4 \)

\[
\text{ads/cft: } \frac{\eta}{s} = \frac{\hbar}{4\pi k_B} \quad \text{Universal value}
\]
Transport Coeff.

Son, Starinets etc.

• Linear response theory: causal Green function $\rightarrow$ trans. coeff.

• AdS/CFT calculates $<JJ>$, $<TT>$ easily.

  $<J> \sim dS/dA$

  $<JJ> \sim ddS/dAdA$
Trans. Coeff.
in Dense matter

X.Ge, Y.Matsuo, F.Shu, SJS, Takuya Tsukioka, arXiv:0806.4460
Y. Matsuo, SJS. S.Takeuchi, T. Tsukioka, C.Yoo (APCTP), archiv:0901.0610

Finite temperature and density
↔ charged black hole

Mode Mixing and decoupling

Transport coefficients from ads/cft
Bulk filling branes and AdS R-N black hole [2007, sjs]

\[ S[g_{mn}, A_m] = \frac{1}{2\kappa^2} \int d^5x \sqrt{-g} \left( R - 2\Lambda \right) - \frac{1}{4e^2} \int d^5x \sqrt{-g} F_{mn} F^{mn}, \]

\[ \frac{l^3}{\kappa^2} = \frac{N_c^2}{4\pi^2}, \quad \frac{l}{e^2} = \frac{N_c N_f}{(2\pi)^2}, \quad \frac{e^2}{\kappa^2} = \frac{N_c}{N_f} \lambda^{-2}. \]

Solution to the eq. of M:

\[ ds^2 = \frac{r^2}{l^2} \left( -f(r)(dt)^2 + \sum_{i=1}^{3} (dx^i)^2 \right) + \frac{l^2}{r^2 f(r)} (dr)^2, \]

\[ A_t = -\frac{Q}{r^2} + \mu, \]

\[ f(r) = 1 - \frac{m l^2}{r^4} + \frac{q^2 l^2}{r^6}, \quad \Lambda = -\frac{6}{l^2}, \]
Thermodynamics
[1998, myers et al]

temperature, entropy, energy

\[ T = \frac{r_+^2 f'(r_+)}{4\pi l^2} = \frac{r_+}{\pi l^2} \left( 1 - \frac{1}{2} \frac{q^2 l^2}{r_+^6} \right) = \frac{1}{2\pi b} \left( 1 - \frac{a}{2} \right), \]

\[ s = \frac{2\pi r_+^3}{\kappa^2 l^3} = \frac{\pi l^3}{4\kappa^2 b^3}, \]

\[ \epsilon = \frac{3m}{2\kappa^2 l^3} = \frac{3l^3}{32\kappa^2 b^4} \left( 1 + a \right), \]

\[ p = \frac{\epsilon}{3}, \]

\[ \mu = \frac{Q}{r_+^2}, \]

\[ \rho = \frac{2Q}{e^2 l^3}, \]

pressure, charge density and chemical potential.
Perturbations around RN

\[ g_{mn} \equiv g_{mn}^{(0)} + h_{mn}, \]

\[ \mathcal{A}_m \equiv A_m^{(0)} + A_m, \]

perturbed Einstein eq.

\[
R_{mn}^{(1)} - \frac{1}{2} g_{mn}^{(0)} R^{(1)} - \frac{1}{2} h_{mn} R^{(0)} + h_{mn} \Lambda = \kappa^2 T_{mn}^{(1)}. \\
R_{mn}^{(1)} = \frac{1}{2} \left( \nabla_k \nabla_m h_n^k + \nabla_k \nabla_n h_m^k - \nabla_k \nabla^k h_{mn} - \nabla_m \nabla_n h \right) \\
R^{(1)} = g^{(0)kl} R_{kl}^{(1)} - h^{kl} R_{kl}^{(0)} \\
= \nabla_k \nabla_l h^{kl} - \nabla_k \nabla^k h - h^{kl} R_{kl}^{(0)}, \\
T_{mn}^{(1)} = \frac{1}{\epsilon^2} \left( - F_{mk}^{(0)} F_{nl}^{(0)} h^{kl} + \frac{1}{2} g_{mn}^{(0)} F_{kp}^{(0)} F_{l}^{(0)p} h^{kl} - \frac{1}{4} h_{mn} F_{kl}^{(0)} F^{(0)kl} \\
+ F_{mk}^{(0)} F_{nk}^{(0)} F_{m}^{(0)k} - \frac{1}{2} g_{mn}^{(0)} F_{kl}^{(0)} F^{(0)kl} \right),
\]
perturbed Maxwell eq.

\[
0 = \nabla_m \left( F^{mn} - F^{(0)}_m k h^{nk} + F^{(0)n}_k h^{mk} + \frac{1}{2} F^{(0)mn} h \right) \\
= \frac{1}{\sqrt{-g^{(0)}}} \partial_m \left\{ \sqrt{-g^{(0)}} \left( g^{(0)mk} g^{(0)nl} (\partial_k A_l - \partial_l A_k) \\
- F^{(0)m}_k h^{nk} + F^{(0)n}_k h^{mk} + \frac{1}{2} F^{(0)mn} h \right) \right\}.
\]

Gauge choice and Fourier mode

\[ h_{\mu\nu}(t, z, r) = \int \frac{d^4 k}{(2\pi)^4} e^{-i\omega t + ikz} h_{\mu\nu}(k, r), \]
\[ A_\mu(t, z, r) = \int \frac{d^4 k}{(2\pi)^4} e^{-i\omega t + ikz} A_\mu(k, r), \]

we choose the momenta which are along the z-direction.
Classification of modes

- vector type: \( h_{xt} \neq 0, \ h_{xz} \neq 0, \ \text{(others)} = 0 \)
  \[
  \left( \text{equivalently,} \ h_{yt} \neq 0, \ h_{yz} \neq 0, \ \text{(others)} = 0 \right)
  \]

- tensor type: \( h_{xy} \neq 0, \ h_{xx} = -h_{yy} \neq 0, \ \text{(others)} = 0 \)

- scalar type: \( h_{tz} \neq 0, \ h_{tt} \neq 0, \ h_{xx} = h_{yy} \neq 0, \ \text{and} \ h_{zz} \neq 0, \ \text{(others)} = 0 \)
Vector type pert.

\[ h_{xt}(x) \neq 0, \quad h_{xz}(x) \neq 0, \quad A_x(x) \neq 0, \quad (\text{others}) = 0. \]

\[ u = r_+^2/r^2 \]

\[ B(u) = \frac{A_x(u)}{\mu} = \frac{l^4}{4Qb^2}A_x(u) \]

\[ h_t^x(r) = g^{(0)xx}h_{xt}(r) = (l^2/r^2)h_{xt}(r) \]

\[ h_z^x(r) = g^{(0)xx}h_{xz}(r) = (l^2/r^2)h_{xz}(r) \]

\[ h_t^x(r) = g^{(0)xx}h_{xt}(r) = (l^2/r^2)h_{xt}(r) \]

\[ 0 = h_t^{xx''} - \frac{1}{u}h_t^{xx'} - \frac{b^2}{u_f} \left( \omega kh_t^x + k^2 h_t^x \right) - 3auB', \]

\[ 0 = kf h_t^{xx'} + \omega h_t^{xx'} - 3awuB, \]

\[ f(u) = (1-u)(1+u-au^2) \]

\[ 0 = h_z^{xx''} + \frac{(u^{-1}f)'}{u^{-1}f} h_z^{xx'} + \frac{b^2}{u_f^2} \left( \omega^2 h_z^x + \omega k h_z^x \right), \]

\[ 0 = B'' + \frac{f'}{f} B' + \frac{b^2}{u_f^2} \left( \omega^2 - k^2 f \right) B - \frac{1}{f} h_t^{xx'}, \]
scalar type pert.

\[ h_{tt} \neq 0, \ h_{tz} \neq 0, \ h_{xx} = h_{yy} \neq 0, \text{ and } h_{zz} \neq 0, \ (\text{others}) = 0 \]

def. new variables:

\[
\begin{align*}
        h'_t &= g^{(0)tt}h_{tt} = -\frac{l^2u}{r^2_+f}h_{tt}, & h'_x &= g^{(0)xx}h_{xx} = \frac{l^2u}{r^2_+}h_{xx}, \\
        h'_z &= g^{(0)zz}h_{zt} = \frac{l^2u}{r^2_+}h_{zt}, & h'_z &= g^{(0)zz}h_{zz} = \frac{l^2u}{r^2_+}h_{zz}, \\
        B_\mu &= \frac{A_\mu}{\mu} = \frac{l^4}{4Qb^2}A_\mu, 
\end{align*}
\]

eq. of Motion

\[ u = r^2_+/r^2 \]

\[ f(u) = (1 - u)(1 + u - au^2) \]

and metric perturbations for

\( (t, t), (t, u), (t, z), (u, u), (u, z), (x, x) \) and \((z, z)\) components,
Master equations

\[0 = \Phi''_\pm + \left(\frac{u^{1/2}f}{u^{1/2}f}\right)\Phi'_\pm + V_\pm \Phi_\pm,\]

\[\Phi_\pm \equiv \alpha_\pm \Phi + \beta A,\]

\[\Phi = \frac{1}{4u^{3/4}(4b^2k^2 - 3f')} \left((4b^2k^2 - 3f')h^2_x + 2f(2h'_x + h'_z)\right) \quad \mathcal{A} \equiv 2a \left(-h'_t + 3h''_x - 2B'_t\right)\]

\[\alpha_\pm = C_\pm - 3au, \quad C_\pm = (1 + a) \pm \sqrt{(1 + a)^2 + 4ab^2k^2}.\]

\[\beta = \frac{u^{1/4}}{8},\]

\[V_\pm = \frac{1}{16u^2f^2(4b^2k^2 - 3f')^2} \left\{ 16b^2\omega^2u(4b^2k^2 - 3f')^2 \right.\]

\[-4uf(4b^2k^2 - 3f')(16b^2k^2(2b^2k^2 - C_\pm u + 3au) - 4f'(2b^2k^2 + 3C_\pm u - 9au^2) - 3(f')^2)\]

\[+ f^2\left\{ 16(b^4k^4 + 12C_\pm b^2k^2u \right.\]

\[-108ab^2k^2u^2 + 54C_\pm au^3 - 162a^2u^4)\]

\[-24f'(b^2k^2 - 6C_\pm u - 18au^2) + 9(f')^2 \}\}. \quad (4.6)\]
Sound pole

\[ \Phi_\pm = H_\pm \widetilde{\Phi}_\pm, \quad H_\pm = \begin{cases} 
    u^{-3/4} & \text{for } \Phi_+ \\
    u^{1/4} & \text{for } \Phi_- \\
    \frac{u^{1/4}}{(1 + a) - \frac{3}{2}au} &
\end{cases} \]

\[ 0 = \ddot{\Phi}_\pm + \frac{(H_\pm u^{1/2})'}{H_\pm u^{1/2}} \ddot{\Phi}_\pm + \ddot{V}_\pm \ddot{\Phi}_\pm, \]

\[ \ddot{V}_\pm(u, \omega, k) \equiv V_\pm(u, \omega, k) - V_\pm(u, 0, 0). \]

Infalling BC at horizon

\[ \tilde{\Phi}_+(u) = (1 - u)\nu F_+(u) \quad \nu = -i \frac{\omega}{4\pi T}, \]

perturbative solution near boundary

\[ F_+(u) = F_{+0}(u) + \omega F_{+1}(u) + k^2 G_{+1}(u) + \omega^2 F_{+2}(u) + \mathcal{O}(\omega^3, \omega k^2), \]

\[ F_{+0}(u) = C, \quad \text{(const.)} \]

\[ F_{+1}(u) = \frac{iC_b}{2(2 - a)} \left\{ \log (1 + u - au^2) - \frac{6K_1(u)}{\sqrt{1 + 4a}} \right\}, \]

\[ G_{+1}(u) = \frac{2}{3} C_b \left\{ \frac{K_1(u)}{\sqrt{1 + 4a}} - \frac{1}{(1 + a)u} \right\}, \]

\[ C = \frac{1}{2(k^2 - 3\omega^2)} \left\{ 3ak^2(B_x)^0 + 3ak\omega(B_x)^0 + (1 + a) \left( -k^2(h_i^0) + 2k\omega(h_i^0) + (k^2 - \omega^2)(h_r^0) + \omega^2(h_r^0) \right) \right\}. \]
On-shell action

\[ S = S_0 + S_{GH} + S_{ct} \]

\[ \frac{l^3}{32\kappa^2 b^4} \int \frac{d^4k}{(2\pi)^4} \left\{ \frac{1}{u} h_i^zh_i^z' + \frac{f}{u} h_i^zh_x^z' + \frac{f}{u} h_i^zh_{i'}^z + \frac{f}{2u} h_{i}^zh_{i'}^z \\
+ \frac{f}{u} h_x^zh_{i'}^z + \frac{f}{u} h_x^zh_{i'}^z + \frac{f}{u} h_x^zh_{i'}^z + \frac{f}{u} h_x^zh_{i'}^z \\
+ \frac{3}{4u^2} \left( f - \sqrt{f} \right) (h_{i'}^z)^2 + \frac{1}{4u^2} \left( 3f - uf' - 3\sqrt{f} \right) (h_{i'}^z)^2 \\
- \frac{3}{u^2f} \left( f - \sqrt{f} \right) (h_{i'}^z)^2 - \frac{1}{2u^2} \left( 6f - uf' - 6\sqrt{f} \right) h_i^zh_{i'}^z \\
- \frac{1}{4u^2} \left( 6f - uf' - 6\sqrt{f} \right) h_i^zh_{i'}^z - \frac{1}{u^2} \left( 3f - uf' - 3\sqrt{f} \right) h_x^zh_{i'}^z \\
+ 3a \left( B_iB_i' - fB_iB_i' + \frac{1}{2} B_i^2 + B_i^2 - B_i^2 - \frac{1}{2} B_i^2 \right) \right\} \bigg|_{u=0}. \]

\[ \frac{1}{u} h_i^z \rightarrow \frac{3}{k^2 - 3\omega^2} \left\{ -3ak \left( k(B_i)^0 + \omega(B_z)^0 \right) \\
+ (1 + a) \left( k^2(h_i)^0 - 2\omega k(h_i)^0 - \omega^2(2(h_x)^0 + (h_z)^0) \right) \right\}, \]

Others are similar.
Correlation functions

### Table 1: \( G_{**} \)

<table>
<thead>
<tr>
<th></th>
<th>( tt )</th>
<th>( xx )</th>
<th>( zz )</th>
<th>( tz )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( tt )</td>
<td>(3(5k^2 - 3\omega^2))</td>
<td>(6(k^2 + \omega^2))</td>
<td>(3(k^2 + \omega^2))</td>
<td>(24(k^2 + \omega^2))</td>
</tr>
<tr>
<td>( xx )</td>
<td>---</td>
<td>(16\omega^2)</td>
<td>(2(k^2 + \omega^2))</td>
<td>(16k\omega)</td>
</tr>
<tr>
<td>( zz )</td>
<td>---</td>
<td>---</td>
<td>(-k^2 + 7\omega^2)</td>
<td>(8k\omega)</td>
</tr>
<tr>
<td>( tz )</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>(4(k^2 + 9\omega^2))</td>
</tr>
</tbody>
</table>

### Table 4: \( G_{**} \)

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<th></th>
<th>( tx )</th>
<th>( zx )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( tx )</td>
<td>(k^2)</td>
<td>(-k)</td>
</tr>
<tr>
<td>( zx )</td>
<td>---</td>
<td>(\omega^2)</td>
</tr>
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</table>

Table 3: \( G_{**} \)

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<th>( z )</th>
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<tr>
<td>( t )</td>
<td>(k^2)</td>
<td>(k\omega)</td>
</tr>
<tr>
<td>( z )</td>
<td>---</td>
<td>(\omega^2)</td>
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</table>

Table 6: \( G_{**} \)

<table>
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<th></th>
<th>( x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>(i\omega)</td>
</tr>
</tbody>
</table>

Diffusion constant:

\[
D_A = \frac{(2 + a)b}{2(1 + a)} \quad D_H = \frac{b}{2(1 + a)}
\]

### Table 7: \( G_{**} \)

<table>
<thead>
<tr>
<th></th>
<th>( xy )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( xy )</td>
<td>(i\omega + bk^2)</td>
</tr>
</tbody>
</table>
conductivity

Kubo formula,

$$
\sigma \equiv - \lim_{\omega \to 0} \frac{e^2}{3\omega} \operatorname{Im}(\delta^{ij}G_{ij}(\omega, k = 0)),
$$

$$
= \left( \frac{l}{e^2} \right) \frac{\pi e_E^2 (2 - a)}{2(1 + a)^2} T.
$$

charge susceptibility

$$
\Xi \equiv \frac{1}{T \text{ (volume)}} \left\langle Q^2 \rightangle.
$$

$$
= \int \frac{d\omega}{2\pi} \left( - \operatorname{Im}(G_{t t}(\omega, k \to 0)) \right) n_b(\omega),
$$

$$
- \operatorname{Im}(G_{t t}(\omega, k)) = \frac{4\pi^2 l T^2}{e^2(1 + a)(2 + a)} \left( \frac{\omega D_A k^2}{\omega^2 + (D_A k^2)^2} \right)
$$

approaches to $2\pi \delta(\omega)$ for $k \to 0$. 
Einstein Relation

charge susceptibility

\[ \Xi = \left( \frac{l}{e^2} \right) \frac{4\pi^2}{(1 + a)(2 + a)} T^2. \]

Einstein Relation holds exactly

\[ \sigma/(eE^2\Xi) = D_A \]
$a = 2 - \frac{4}{1 + \sqrt{1 + 4(\mu/T)^2}}, \quad b = \left(\frac{1}{\pi T}\right) \frac{1}{1 + \sqrt{1 + 4(\mu/T)^2}},$

**Behaviour of Transport coefficients.**

**Figure 1:** $TD_A$ vs. $\mu/T$

**Figure 2:** $TD_H$ vs. $\mu/T$

**Figure 3:** $\sigma/(Tl_E^2/e^2)$ vs. $\mu/T$.

**Figure 4:** $\Xi/(T^2l^2/e^2)$ vs. $\mu/T$ : Notice the rapid change between two finite values as $T$ runs from 0 to $\infty$. 
Jet-Quenching

- Energy Loss Problem.
Holography of radiation

SJS with Zahed(hep-th/0407215, PLB)

- gluon propagation at boundary v.s null geodesic along the path passing the center.

- We get point–sphere correspondence.

\[ \theta + \psi = \pi / 2 \]
Mikhailov (hep-th/0305196) calculated energy of ripple along the string from N to S passing through the center.

\[ E = \frac{\sqrt{\lambda}}{2\pi} \int dt \frac{r^2 - (v \times a)^2}{(1 - v^2)^2}, \]

with \( v = \frac{dx}{dt}, a = \frac{d^2 x}{dt^2} \) 

Lienard formula for the radiation apart from square root.
Holography of radiation in Black hole background

Maximal propagation distance = $\frac{1}{\pi T}$

P will be absorbed into the BH or never get to it according to the observer.
The dual picture to this is stopping at
Idea of Early Thermalization

(hep-th/0511199)

Dual Q: why BH is easily formed in ads?

Scattering and particle creation in the boundary has its dual creation at the bulk. It is peaked at some height and fall.

The point is that the proper time to fall to the center in ads is the same no matter where you start.
Falling in AdS and time-focusing effect

It enhances black hole formation in AdS compared with flat spacetime.
Conclusion

• String theory may be useful for the New experiment. AdS/RHIC, Ads/QCD, ads/CDM etc. etc.

• String theory may also be useful for Hadronic dense matter.